The Electrodynamic Origin of the Force of Inertia ($F = m_i a$)—Part 3

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Abstract. A review of Newton's *Principia* [3] shows his dependence on his Existence Theorem for absolute space and time in order to explain the force of inertia and the centrifugal force in terms of absolute coordinates. A review of the history of Einstein's General Theory of Relativity reveals his failure to establish the basis of inertia and the centrifugal force in terms of relative coordinates as Mach [5] had envisioned. In this work the force of inertia, including the centrifugal force, is derived from the universal electrodynamics force law based on relative coordinates. From the electrodynamics perspective the inertial force is an average residual force between vibrating neutral electric dipoles consisting of atomic electrons vibrating with respect to protons in the nucleus of atoms. The inertial mass is derived and shown to be equal to the derived gravitational mass resulting from the same universal force law. The vibrational mechanism for both gravitational and inertial mass causes the magnitude of both masses to decay over time. The derived electrodynamics inertial force has a second term, a non-radial **R x (R x A)** term, which describes certain observed non-Newtonian inertial gyroscopic motions. Arguments are made that this derived law of inertia is superior to both Newton's Law of Inertia (**F** = m**a**) and Einstein's field equations of General Relativity Theory, because (1) it is properly based on local contact forces instead of unphysical action-at-a-distance forces, (2) it is based on forces between finite-size particles instead of imaginary point particles, (3) it is based on relative coordinates instead of fictitious absolute space coordinates, (4) it is derived from a universal force law, (5) it explains the centrifugal force as a piece of the inertial force, (6) it is simpler and does not need mass as a fundamental quantity, (7) it explains the apparent equivalence of gravitational and inertial mass, (8) it contains a new non-radial **R x (R x A)** term that describes additional observed phenomena not previously explained by any theory of inertia, and (9) it contains relativistic type v/c corrections for high velocity.

Centrifugal Force Is an Inertial Force. For this calculation it is convenient to note the following definitions:

$$\begin{aligned}
\vec{R} &= \vec{r}_1 - \vec{r}_2 = (x_1 - x_2)\hat{x} + (y_1 - y_2)\hat{y} + (z_1 - z_2)\hat{z} \\
\vec{V} &= \frac{d\vec{R}}{dt} = \vec{v}_1 - \vec{v}_2 = (\dot{x}_1 - \dot{x}_2)\hat{x} + (\dot{y}_1 - \dot{y}_2)\hat{y} + (\dot{z}_1 - \dot{z}_2)\hat{z} \qquad (19) \\
\vec{A} &= \vec{a}_1 - \vec{a}_2 = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2} = (\ddot{x}_1 - \ddot{x}_2)\hat{x} + (\ddot{y}_1 - \ddot{y}_2)\hat{y} + (\ddot{z}_1 - \ddot{z}_2)\hat{z}
\end{aligned}$$

where the separation distance between two charges is R, the magnitude of the relative velocity is V, and magnitude of relative acceleration is A which are given by

$$R = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$V = \frac{dR}{dt} = \frac{(x_1 - x_2)(\dot{x}_1 - \dot{x}_2) + (y_1 - y_2)(\dot{y}_1 - \dot{y}_2) + (z_1 - z_2)(\dot{z}_1 - \dot{z}_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}} = \hat{R} \cdot \vec{V}$$

$$A = \frac{dV}{dt} = \frac{d^2R}{dt^2} = \frac{(\dot{x}_1 - \dot{x}_2)(\dot{x}_1 - \dot{x}_2) + (\dot{y}_1 - \dot{y}_2) + (\dot{y}_1 - \dot{y}_2) + (\dot{z}_1 - \dot{z}_2)(\dot{z}_1 - \dot{z}_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$

$$- \frac{((x_1 - x_2)(\dot{x}_1 - \dot{x}_2) + (y_1 - y_2) + (\dot{y}_1 - \dot{y}_2) + (z_1 - z_2)(\dot{z}_1 - \dot{z}_2))^2}{\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\right)^3}$$

$$+ \frac{(x_1 - x_2)(\ddot{x}_1 - \ddot{x}_2) + (y_1 - y_2)(\ddot{y}_1 - \ddot{y}_2) + (z_1 - z_2)(\ddot{z}_1 - \ddot{z}_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}} \qquad (20)$$

$$= \frac{\vec{V}^2 - (\hat{R} \cdot \vec{V})^2 + \vec{R} \cdot \vec{A}}{R} \quad \text{where } \hat{R} = \frac{\vec{R}}{R}$$

From equation (20) the acceleration term is actually three terms. For acceleration A and velocity V perpendicular to R the acceleration term becomes just the normal centrifugal force $A = V^2/R$. For velocity V parallel to R then $A = \hat{\mathbf{R}} \cdot \mathbf{A}$. For anything in between all three terms must be taken into account.

Note that this approach explains Newton's law of inertia and the centrifugal force in terms of relative coordinates. This was the goal of Mach and Einstein, but it was never satisfactorily attained by them.

Computation of the Second Acceleration Term. Equation (8) of Part 2 has a second acceleration term proportional to **R** x (**R** x **A**) as shown below. This term is unknown in classical mechanics, although it has been seen in many gyroscope experiments such as those of Eric Laithwait [6]. This term will lead to a corkscrew type of motion just like the electrodynamic case of particle beams and the gravitational case of planetary orbits. It will be one more piece of evidence that the electrodynamics force law is indeed the "universal" force law.

$$\vec{F}(\vec{R},\vec{V},\vec{A}) = \frac{qq'\frac{2}{c^2}\vec{A}}{\left(1-\beta^2\sin^2\theta\right)^{1/2}} - \frac{qq'\left(1-\beta^2\right)\vec{R}\times\left(\vec{R}\times\frac{\vec{A}}{c^2}\right)}{\left[\vec{R}^2 - \frac{\left\{\vec{R}\times\left(\vec{R}\times\beta\right)\right\}^2}{\vec{R}^2}\right]^{3/2}} \tag{8}$$

$$= qq'\frac{2}{c^2}\vec{A}\left[1+\frac{\beta^2\sin^2\theta}{2}+\ldots\right] - \frac{q\vec{q}}{\vec{R}}\hat{R}\times\left(\hat{R}\times\frac{\vec{A}}{c^2}\right)\left(1-\beta^2\right)\left[1+\frac{3}{2}\beta^2\sin^2\theta+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{2}\beta^4\sin^4\theta\ldots\right]$$

$$= qq'\frac{2}{c^2}\vec{A}\left[1+\frac{\beta^2}{2}-\frac{\beta^2}{2}\cos^2\theta+\ldots\right]$$

$$-\frac{qq'}{R}\hat{R}\times\left(\hat{R}\times\frac{\vec{A}}{c^2}\right)\left[1+\frac{\beta^2}{2}-\frac{3}{2}\beta^2\cos^2\theta-\frac{3}{8}\beta^4-\frac{9}{4}\cos^2\theta+\frac{15}{8}\beta^4\cos^4\theta+\ldots\right]$$

The force terms for the second acceleration term to order β^4 are given in equation (8). For the velocity terms in the [ ] of the expressions for the force, consider the case $\beta_2 = \beta_1$. In this case only the $A_1\omega_1$ terms are left giving

$$\vec{F}_{2+1-} = -\frac{2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\times$$

$$\left[1+\left(A_1\omega_1\sin\left(\omega_1 t+\varphi_1\right)\right)^2\frac{1-3\cos^2\theta}{2}-\left(A_1\omega_1\sin\left(\omega_1 t+\varphi_1\right)\right)^4\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right)\right]$$

$$\vec{F}_{2+1+} = \frac{2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\,[1] \tag{21}$$

One can see that the sum of the first terms in the [ ] of the two forces is just 0. Thus the total force is

$$\vec{F}\left(R,\theta,\varphi,A_1,\omega_1,t\right) = \vec{F}_{2+1+}+\vec{F}_{2+1-} = -\frac{2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\times \tag{22}$$

$$\left[\left(A_1\omega_1\sin\left(\omega_1 t+\varphi_1\right)\right)^2\frac{1-3\cos^2\theta}{2}-\left(A_1\omega_1\sin\left(\omega_1 t+\varphi_1\right)\right)^4\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right)\right]$$

Now the integrals of equation (22) can be evaluated. Note that the integral over θ for the $(1 - 3\cos^2\theta)/2$ term averages to zero.

$$\frac{1}{\pi}\int_0^\pi \sin\theta\, d\theta\frac{\left(1-3\cos^2\theta\right)}{2} = \frac{-1}{\pi}\int_0^\pi d\cos\left(\frac{1-3\cos^2\theta}{2}\right) = \frac{-1}{2\pi}\left(\cos\theta-\cos^3\theta\right)\Big|_0^\pi \tag{23}$$

$$= \frac{-1}{2\pi}\left(-1-1-(-1)+1\right) = 0$$

Thus

$$\vec{F}_{2+1-} = -\frac{2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\left[1+\left(\vec{\beta}_2-\vec{\beta}_1\right)^2\ \frac{\left(1-3\cos^2\theta\right)}{2}-\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right)+\ldots\right] \quad (24)$$

$$\vec{F}_{2+1+} = \frac{2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\left[1+\left(\vec{\beta}_2-\vec{\beta}_1\right)^2\ \frac{\left(1-3\cos^2\theta\right)}{2}-\left(\vec{\beta}_2-\vec{\beta}_1\right)^4\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right)+\ldots\right]$$

where

$$\vec{F} = \frac{1}{T_1}\int_0^{T_1}dt\ \frac{1}{2\pi}\int_0^{\pi}d\varphi_1\ \frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\ \vec{F}(R,\theta,\varphi,A_1,\omega_1,t) \quad (25)$$

$$= \frac{1}{T_1}\int_0^{T_1}dt\ \frac{1}{2\pi}\int_0^{\pi}d\varphi_1\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\left(\frac{-2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\right)\left(\frac{A_1^4\,\omega_1^4}{c^4}\right)\sin^4(\omega_1 t+\varphi_1)\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right)$$

$$= \frac{1}{T_1}\int_0^{T_1}dt\ \frac{1}{2\pi}\int_0^{\pi}d\varphi_1\left(\frac{-2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\right)\left(\frac{A_1^4\,\omega_1^4}{c^4}\right)\sin^4(\omega_1 t+\varphi_1)\left(\frac{3}{2\pi}\right)$$

$$= \frac{\omega_1}{2\pi}\int_0^{2\pi/\omega_1}dt\ \left(\frac{-2e^2\,\hat{R}\times\left(\hat{R}\times\vec{A}\right)}{Rc^4}\right)\left(\frac{3}{8}\right)\left(\frac{A_1^4\,\omega_1^4}{c^4}\right)\left(\frac{3}{2\pi}\right)$$

$$= \left(\frac{-9e^2}{8\pi c^4}\right)\ \frac{A_1^4\,\omega_1^4}{Rc^4}\ \hat{R}\times\left(\hat{R}\times\vec{A}\right) = \frac{27}{32}\ \frac{A_1^2\,\omega_1^2}{Rc^4}\,m_{i1}\hat{R}\times\left(\hat{R}\times\vec{A}\right)$$

and

$$\frac{1}{\pi}\int_0^{\pi}\sin\theta\,d\theta\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right) = \frac{1}{\pi}\int_{-1}^{+1}d\cos\theta\left(\frac{3}{8}+\frac{9}{4}\cos^2\theta-\frac{15}{8}\beta^4\cos^4\theta\right) \quad (26)$$

$$= \frac{1}{\pi}\left(\frac{3}{8}\cos\theta+\frac{9}{4}\frac{\cos^3\theta}{3}-\frac{15}{8}\frac{\cos^5\theta}{5}\right)\Bigg|_{-1}^{+1}$$

$$= \frac{1}{\pi}\left(\frac{3}{8}\,2+\frac{9}{4}\,\frac{2}{3}-\frac{15}{8}\,\frac{2}{5}\right) = \frac{3}{2\pi}$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi \sin^4\left(\omega t+\varphi\right) \qquad (27)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\varphi \left[\sin^4 \omega t \cos^4 \varphi + 2\sin^3 \omega t \cos^3 \varphi \cos \omega t \sin \varphi + \sin^2 \omega t \cos^2 \omega t \sin^2 \varphi \cos^2 \varphi\right.$$

$$\left. + 4\cos^2 \omega t \sin^2 \omega t \sin^2 \varphi \cos^2 \varphi + 2\cos^3 \omega t \sin \omega t \sin^3 \varphi \cos \varphi + \cos^4 \omega t \sin^4 \varphi \right]$$

$$= \frac{1}{2\pi}\left[\frac{3\pi}{4}\sin^4 \omega t \; \pi \cos^2 \omega t \sin^2 \omega t + \frac{2\pi}{4}\sin^2 \omega t \cos^2 \omega t + \frac{3\pi}{4}\cos^4 \omega t\right]$$

$$= \frac{3}{8}\left[\sin^4 \omega t + 2\cos^2 \omega t \; \sin^2 \omega t + \cos^4 \omega t\right] = \frac{3}{8}\left[\sin^2 \omega t + \cos^2 \omega t\right]^2 + \frac{3}{8}\left[1\right]^2 = \frac{3}{8}$$

Note that the odd powers of $\sin\varphi$ and $\cos\varphi$ in equation (27) above integrate to zero.

Thus the full inertial force law is given by

$$\vec{F}_1 = m_{i1}\vec{A} - \frac{27}{32}\frac{A_1^2\,\omega_1^2}{R c^4} m_{i1}\hat{R}\times\left(\hat{R}\times\vec{A}\right) \qquad (28)$$

Summary of Part 3. A new classical electrodynamics inertial force law was derived from a local contact type universal electrodynamics force law for finite size particles. In this force law mass is not a fundamental quantity of nature, but merely a common grouping of electromagnetic factors. The first acceleration term of the electrodynamics force, of order β^2, gives rise to Newton's second law for non-relativistic velocities. The second term, of order β^4, is a new non-radial **R** x **(R** x **A)** term that gives rise to a corkscrew type of spiraling inertial motion. The strength of the second term is normally much less than that of the first due to the $A^2\omega^2/Rc^4$ factor. The second term, however, can be quite large for fast spinning large gyroscopes where $A^2\omega^2/Rc^4$ gets large and causes unexpected behavior as publicly demonstrated by Eric Laithwaite [6].

This derived law of inertia appears to be superior to both Newton's Law of Inertia (**F** = *m***a**) and Einstein's field equations of General Relativity Theory, because (1) it is properly based on local contact forces instead of unphysical action-at-a-distance forces, (2) it is based on forces between finite-size particles instead of imaginary point particles, (3) it is based on relative coordinates instead of fictitious absolute space coordinates, (4) it is derived from a universal force law, (5) it explains the centrifugal force as a piece of the inertial force, (6) it is simpler and does not need mass as a fundamental quantity, (7) it explains the apparent equivalence of gravitational and inertial mass, (8) it contains a second non-radial **R** x **(R** x **A)** term that describes additional observed phenomena not previously explained, and (9) it contains relativistic type *v/c* corrections for high velocity.

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