Derivation of the Universal Force Law—Part I

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Abstract. A new universal electromagnetic force law for real finite-size elastic charged particles is derived by solving simultaneously the fundamental empirical laws of classical electrodynamics, *i.e.* Gauss's laws, Ampere's generalized law, Faraday's law, and Lenz's law assuming Galilean invariance. This derived version of the electromagnetic force law incorporates the effects of the self-fields of real finite-size elastic particles as observed in particle scattering experiments. It can account for gravity, inertia, and relativistic effects including radiation. The non-radial terms of the force law explain the experimentally observed curling of plasma currents, the tilting of the orbits of the planets with respect to the equatorial plane of the sun, and certain inertial gyroscope motions. The derived force law satisfies Newton's third law, conservation of energy and momentum, conservation of charge, and Mach's Principle. The mathematical properties of equations for the fundamental empirical laws and also Hooper's experiments showing that the fields of a moving charge move with the charge require that the electrodynamic force be a contact force based on field extensions of the charge instead of an action-at-a-distance force. The Lorentz force is derived from Galilean invariance. The most general form of the force law, derived using all the higher order terms of the Galilean transformation, is assumed to be exact for all phenomena on all size scales. Arguments are given that this force law is superior to all previous force laws, *i.e.* relativistic quantum electrodynamic, gravitational, inertial, strong interaction and weak interaction force laws.

Introduction. Wilhelm Weber (1804-1890) published his action-at-a-distance force theory for electrodynamics in 1846 and 1848 following in the footsteps of Newton. His theory was designed to agree with the following notions:

1. Coulomb's law for the force between static charges
2. Ampere's law for the force between current elements
3. Faraday's law of electromagnetic induction
4. Newton's third law
5. Conservation of energy
6. Point-like charged particles
7. Action-at-a-distance electromagnetic forces between point particles
8. Constant forces

Weber formulated a velocity and acceleration dependent force based upon careful experimental analysis. He postulated a velocity-dependent potential, U_W, that gives rise to this force between two moving charges q at r and q' at r' as

$$U_W(R,V) = \frac{qq'}{R}\left(1 - \frac{\vec{V}^2}{c^2}\right) \quad (1)$$

$$\text{where } R = |\vec{r}' - \vec{r}|,\ V = \frac{dR}{dt}, A = \frac{dV}{dt}$$
$$\text{and } c = \text{velocity of light}$$

For V = dR/dt = 0 the potential U_W in equation (1) becomes just the Coulomb potential. (Note Weber's original constant *c* was a measure of the mechanical to the electrodynamic current density equal to *c* times the square root of 2 where this *c* is the commonly used value for *c* today [26]).

For conservative systems the forces are derivable from a scalar potential, *i.e.*

$$U\left(\vec{R},\vec{V},\vec{A},\dot{\vec{A}},\ldots\right) = -\vec{F}\left(\vec{R},\vec{V},\vec{A},\dot{\vec{A}},\ldots\right)\cdot\vec{R}$$
$$\frac{dU}{dt} = -\vec{F}\cdot\frac{d\vec{R}}{dt} - \frac{d\vec{F}}{dt}\cdot\vec{R} = -\vec{F}\cdot\vec{V} - \frac{d\vec{F}}{dt}\cdot\vec{R} \quad (2)$$

For a constant force dF/dt = 0 , then

$$\frac{dU}{dt} = -\vec{F}\cdot\vec{V} \quad (3)$$

Thus the Weber force F_W on charge q at r due to q' at r' is obtained from equation (1) using equation (3). For this calculation it is convenient to note the following definitions:

$$\vec{R} = \vec{r} - \vec{r}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$
$$\vec{V} = \frac{d\vec{R}}{dt} = \vec{v} - \vec{v}' = (\dot{x} - \dot{x}')\hat{x} + (\dot{y} - \dot{y}')\hat{y} + (\dot{z} - \dot{z}')\hat{z} \quad (4)$$
$$\vec{A} = \vec{a} - \vec{a}' = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2} = (\ddot{x} - \ddot{x}')\hat{x} + (\ddot{y} - \ddot{y}')\hat{y} + (\ddot{z} - \ddot{z}')\hat{z}$$

where the separation distance between two charges is R, the magnitude of the relative velocity is V, and magnitude of relative acceleration is A which are given by

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$
$$V = \frac{dR}{dt} = \frac{(x - x')(\dot{x} - \dot{x}') + (y - y')(\dot{y} - \dot{y}') + (z - z')(\dot{z} - \dot{z}')}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = \hat{R}\cdot\vec{V}$$
$$A = \frac{dV}{dt} = \frac{d^2R}{dt^2} = \frac{(\dot{x} - \dot{x}')(\dot{x} - \dot{x}') + (\dot{y} - \dot{y}')(\dot{y} - \dot{y}') + (\dot{z} - \dot{z}')(\dot{z} - \dot{z}')}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$
$$- \frac{\left((x - x')(\dot{x} - \dot{x}') + (y - y')(\dot{y} - \dot{y}') + (z - z')(\dot{z} - \dot{z}')\right)^2}{\left(\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}\right)^3}$$
$$+ \frac{(x - x')(\ddot{x} - \ddot{x}') + (y - y')(\ddot{y} - \ddot{y}') + (z - z')(\ddot{z} - \ddot{z}')}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} \quad (5)$$
$$= \frac{\vec{V}^2 - \left(\hat{R}\cdot\vec{V}\right)^2 + \vec{R}\cdot\vec{A}}{R} \quad \text{where } \hat{R} = \frac{\vec{R}}{R}$$

Note:

$$\frac{d}{dt}\left(\frac{1}{R}\right) = -\frac{dR\,/\,dt}{R^2} = -\frac{\hat{R}\cdot\vec{V}}{R^2}$$

$$\begin{aligned}\frac{d\vec{V}^2}{dt} &= \frac{d}{dt}\Big[(\dot{x}-\dot{x}')^2 + (\dot{y}-\dot{y}')^2 + (\dot{z}-\dot{z}')^2\Big] \\ &= 2\Big[(\dot{x}-\dot{x}')(\ddot{x}-\ddot{x}') + (\dot{y}-\dot{y}')(\ddot{y}-\ddot{y}') + (\dot{z}-\dot{z}')(\ddot{z}-\ddot{z}')\Big] \quad (6)\\ &= 2\vec{V}\cdot\vec{A}\end{aligned}$$

Thus Weber's force F_W is obtained from equations (1), (3), and (6) as

$$\vec{V}\cdot\vec{F}_W = -\frac{dU_W}{dt} = \frac{qq'}{R^3}\left[\left(1-\frac{\vec{V}^2}{c^2}\right)\vec{R} + \frac{2\vec{R}^2}{c^2}\vec{A}\right]\cdot\vec{V} \quad (7)$$

Note Weber's force F_W clearly obeys Newton's third law.

Assuming that the work with a constant force done to move one charged particle away from another is the change in the potential, one may write using Newton's third law ($F = -F'$) and then Newton's second law ($F = ma$ and $F' = ma'$)

$$-\frac{dU}{dt} = \vec{V}\cdot\vec{F} = (\vec{v}-\vec{v}')\cdot\vec{F} = \vec{v}\cdot\vec{F} + \vec{v}'\cdot\vec{F}' = \vec{v}\cdot m\vec{a} + \vec{v}'\cdot m'\vec{a}' = \frac{d}{dt}\left(\frac{m\vec{v}^2}{2} + \frac{m'\vec{v}'^2}{2}\right) \quad (8)$$

Integrating over time yields the conservation of energy

$$E = U + \frac{m\vec{v}^2}{2} + \frac{m'\vec{v}'^2}{2} \quad (9)$$

where E, the total energy, is a constant of integration.

In this way Weber showed that his force satisfied conservation of energy. Weber proved the equivalence of his law to that of Ampere [l] and derived Ampere's law from his velocity-dependent potential.

Despite the agreement of Weber's force law with experiment and other theories in the list above, a fatal objection was raised by Helmholtz [2]. According to Whittaker [3], Helmholtz objected to the negative sign of the second term in equation (7). This term implies "that a charge behaves somewhat as if its mass were negative, so that in certain circumstances its velocity might increase indefinitely under the action of a force opposed to the motion" [3]. As a result of this objection, Weber's potential and his force law were discredited and abandoned.

Attempts to Improve Weber's Force Law. Phipps [4] has tried to modernize Weber's potential and his force law by postulating a relativistic-like velocity dependence, *i.e.*

$$U_{Phipps} = \frac{qq'}{R}\sqrt{1-\frac{\vec{V}^2}{c^2}} = \frac{qq'}{R}\left[1-\frac{\vec{V}^2}{2c^2}+\ldots\right] \quad (10)$$

$$\vec{F}_{Phipps} = \frac{qq'\vec{R}}{|\vec{R}|^3}\left\{\sqrt{1-\frac{\vec{V}^2}{c^2}} + \frac{1}{c^2}\vec{R}\cdot\vec{A}/\sqrt{1-\frac{\vec{V}^2}{c^2}}\right\} \quad (11)$$

Phipps' notion is that Weber's potential and force law were just a first order approximation to the relativistic expression. This revised version of Weber's force law is in agreement with the experimental observation in the transverse direction (*i.e.* perpendicular to the direction of motion), the laws 1-5 in the list above, and is free of Helmholtz's objection. This new version of the Weber force law fails, however, in that it does not predict the experimentally observed transverse and longitudinal angular dependence of the relativistic force between moving charges in linear and circular accelerators [9b *p.* 555 or 9c *p.* 560] as given in equation (12). Also it fails to obtain the second or non-radial term with (R·V){R×(R×V/c)}.

$$\vec{F}_{Experiment} = \frac{qq'\hat{R}}{|\vec{R}|^2}\frac{\left[1-\frac{V^2}{c^2}\right]}{\sqrt{1-\frac{V^2}{c^2}\sin^2\theta}} + \frac{qq'}{|\vec{R}|^2}\frac{\left[1-\frac{V^2}{c^2}\right]\left(\hat{R}\cdot\frac{\vec{V}}{c}\right)\left(\hat{R}\times\left(\hat{R}\times\frac{\vec{V}}{c}\right)\right)}{\left[1-\frac{V^2}{c^2}\sin^2\theta\right]^{3/2}} \quad (12)$$

Wesley [5] proposed a variation of Phipps' version of the Weber potential in terms of the absolute velocities of the individual charges, *i.e.*

$$U_{Wesley} = \frac{qq'}{R}\sqrt{1-\frac{\vec{v}^2}{c^2}}\sqrt{1-\frac{\vec{v}'^2}{c^2}}\left\{1-\frac{(\vec{R}\cdot\vec{v})(\vec{R}\cdot\vec{v}')}{c^2R^2}\right\} \quad (13)$$

This potential reduces to the Weber potential in the non-relativistic limit and has limit velocities given by $|v| \le c$, $|v'| \le c$ and $|V| \le 2c$. The force derived from this potential for velocities approaching c behaves the same as the original Weber theory. It too fails to agree with accelerator experiments as given by equation (12).

The purpose of the present work is to simultaneously solve the fundamental empirical laws of electrodynamics to obtain a more general expression of the electrodynamic force law that has the relativistic-like velocity and acceleration dependence with the correct angular dependence so that the electrodynamic forces within real finite-size elastic elementary particles can be calculated [6,7]. Also this derived electrodynamic force should naturally include the radiation and radiation reaction terms.

Problems With Various Formulations of Electrodynamics. In his paper Phipps [4] notes a problem with Einstein's theory of special relativity as integrated with electrodynamics. The problem is that Einstein's relativity theory is internally self-contradicting in that it is not a true relativistic theory. Instead of being dependent upon the relative velocities of the bodies directly involved, it is relative to a frame of reference. This is a characteristic of absolutism as represented by Galilean invariance and not a true characteristic of relativism.

O'Rahilly[8] notes that Weber's claim of the compatibility of his force law with energy conservation is actually a weakness of the law. He reasons that since the force law contains an acceleration term, it must describe radiation. However, if the moving particle is radiating energy, then Weber's law must not conserve energy, because it does not include the radiation reaction terms which depend on dA/dt. Since Weber's law does not contain such terms, the law must be incomplete. This work will satisfy O'Rahilly's criticism.

To these objections above for Maxwell's equations, relativity theory, and Weber's force law the author adds the following:

1. The covariant form of Ampere's law in Maxwell's equations [9a, *pp.* 138 and 379 or 9b, *pp.* 174 and 551 or 9c, *pp.* 179 and 557] is

$$\nabla \times \vec{B}(\vec{r},t) - \frac{1}{c}\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = \frac{4\pi}{c}\vec{J}(\vec{r},t) + \frac{1}{c}\nabla \int \frac{\nabla' \cdot \vec{J}(\vec{r}\,',t')}{|\vec{r}-\vec{r}\,'|} d^3 r' \approx \frac{4\pi}{c}\vec{J}(\vec{r},t) \quad (14)$$

Covariant Approximation

where the last step in (14) is the covariant approximation that neglects the induced field effects due to finite-size particles. Thus the covariant form of electrodynamics based on Maxwell's equations has a point-particle approximation built-in and is not strictly correct.

2. Special relativity theory and Maxwell's equations do not satisfy Mach's principle, but the fundamental equations of electrodynamics can do so through the application of Lenz's law.

3. The fundamental empirical laws of electrodynamics, *i.e.* Gauss's laws, Faraday's law, Ampere's generalized law, and Lenz's law, are based on absolute reference frames subject to Galilean invariance. Relativity theory is based on Lorentz invariance which is logically inconsistent with Galilean invariance. Thus it is logically improper to impress Lorentz invariance on the fundamental laws of electrodynamics which already were assumed to obey a different transformation [9a *pp.* 170-173 or 9b *pp.* 210-213 or 9c *pp.* 208-211].

4. Weber's force law is missing the transverse and longitudinal angular dependence in equation (12) plus the (R·V){R×(R×V/c)} term.

5. Weber's force law is missing the radiation and radiation reaction terms.

6. Weber's force law is based on the illogical and physically unacceptable notion of action-at-a-distance like Newton's gravitational force laws, but experiments show that the fields remain attached to moving charges and exert forces on distant charges due to their tensile strength[12,13,14].

7. Weber's force law is missing the $(V/c)^4$ term responsible for gravitation.

8. Maxwell's equations are based upon the superposition principle which states that static and induced electromagnetic fields are identical, but experimentally they are different [12,13,14].

Derivation of Electrodynamic Force Law for Distributed Particles. In the derivation that follows[10, 11a, 11b] the approach is taken that the general electrodynamic force law should be compatible with the following set of notions which is more complete than that of Weber:

1. Coulomb's law for the force between static charges
2. Ampere's generalized law for the force between current elements
3. Faraday's law of electromagnetic induction
4. Newton's third law
5. Conservation of kinetic and radiation energy
6. Conservation of momentum (radiation reaction, etc.)
7. Mach's principle
8. Galilean invariance
9. Gauss's laws
10. Lenz's law for induction
11. Finite size of charged particles
12. Only contact forces exist in nature
13. Lorentz's force law
14. Newton's Universal Law of Gravitation
15. Newton's Law of Inertia
16. Superposition principle is invalid in electrodynamics
17. Fields of charges remain attached when charges move and have tensile strength

The fundamental equations of electrodynamics are based upon five empirical laws, *i.e.*

Ampere's Law $\vec{B}_i(\vec{r},t) = \vec{v} / c \times \vec{E}_0(\vec{r}\,',t')$

Faraday's Law $\int \vec{E}(\vec{r}\,',t')d\ell = -1/c\, d/dt \int \vec{B}(\vec{r},t) \cdot \hat{n}da$

Gauss's Laws $\int \vec{E}(\vec{r},t) \cdot \hat{n}da = 4\pi q$ *and* $\nabla \cdot \vec{B}(\vec{r},t) = 0$

Lenz's Law $\vec{E}(\vec{r},t) \propto -\vec{E}_0(\vec{r},t)$ *where* $\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \vec{E}_i(\vec{r},t)$

Lorentz's Law $\vec{F} = q\vec{E}(\vec{r},t) - \vec{v}/c \times \vec{B}_i(\vec{r},t)$

Note that both Ampere's law and Faraday's law involve the observer's reference frame and a moving frame of reference that are described by the Galilean transformation $r' = r - vt - \frac{1}{2}at^2 - \ldots$ and $t' = t$.

In the past physicists have willfully discarded the Galilean transformation in favor of the relativistic Lorentz transformation to relate electromagnetic fields in the two frames. This derivation will show that this illogical procedure was totally unnecessary and resulted in the creation of the superfluous theory of special relativity.

In this paper the fundamental empirical equations of electrodynamics are solved simultaneously by the method of substitution using the Galilean transformation. The resulting electric and magnetic fields in the observer's frame of reference will be derived for an elementary particle with an arbitrary finite-size elastic charge distribution of total charge q moving with relative velocity V, acceleration A = dV/dt, *etc.* The resulting electric and magnetic fields will be found to be in agreement with the experimentally observed fields of very high velocity charged particles in accelerator experiments.

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